Coherent feedback control of multipartite quantum entanglement for optical fields

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Coherent feedback control (CFC) of multipartite optical entangled states produced by a nondegenerate optical parametric amplifier is theoretically studied. The features of the quantum correlations of amplitude and phase quadratures among more than two entangled optical modes can be controlled by tuning the transmissivity of the optical beam splitter in the CFC loop. The physical conditions to enhance continuous variable multipartite entanglement of optical fields utilizing the CFC loop are obtained. The numeric calculations based on feasible physical parameters of realistic systems provide direct references for the design of experimental devices.

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I. INTRODUCTION

As well-known light is the optimal conveyor of quantum information due to its high speed and weak interaction with the environment. The entangled optical fields are the necessary resources for performing continuous variable (CV) quantum information [1–3]. Preparing multipartite entanglement of more than two systems and manipulating entangled quantum states are two fundamental problems in quantum information networks. In the past decades, the CV quantum information based on quantum correlations between amplitude and phase quadratures of optical fields has attracted extensive attention [4,5]. To establish a practical CV quantum information network, one of the most primary tasks is to prepare multipartite entangled states of light with a possibly high entanglement degree for reaching the required fidelity of teleporting quantum information [6–9]. A variety of theoretical and experimental achievements in CV quantum information have been presented [10–13]. Degenerate and nondegenerate optical parametric amplifiers (DOPAs and NOPAs) have been widely applied in CV quantum information systems to be the necessary sources of squeezed and entangled states [14,15]. Tripartite Greenberger-Horne-Zeilinger-like (GHZ-like) and quadripartite entangled states of optical fields have been successfully generated by DOPAs and NOPAs and applied for CV quantum information implementations, such as teleportation networks, controlled dense coding, and quantum error correction via telecloning [16–20]. The multipartite entangled states are the basic resources for transmitting quantum information or quantum states among distant stations or nodes in a quantum network, and a high entanglement degree is the elementary requirement for achieving information transferring and processing with high fidelity. In Refs. [17–19] the tripartite and quadripartite entangled states are obtained by splitting or combining squeezed states of light on optical beam splitters. For constructing quadripartite cluster entangled states in Ref. [18] (Ref. [19]), four squeezed states produced by four DOPAs (two NOPAs) are combining by the beam splitter network. To ensure highly coupling efficiency of squeezed states on each beam splitter, these squeezed states should be classically coherent, so all optical parametric amplifiers (OPAs) used in the system have to be pumped by a laser source and to be phase locked during the experiment. For the practical applications in quantum information networks, it is desired to produce multipartite entangled states directly in a more compact device. Recently, Coelho *et al.* presented an elegant experimental achievement on the generation of a three-color entangled state of light, in which the CV quantum entanglement among the pump, the signal, and the idler optical fields of an optical parameter oscillator was demonstrated first [6]. For connecting different physical systems with respective resonance frequencies at the nodes of a quantum network, it is important to produce the multipartite entangled optical fields with different frequencies [6–8]. In 2004 Pfister *et al.* presented a scheme of obtaining multipartite entanglement by the use of a single OPA without the necessity of beam splitters, which opens a hopeful avenue to prepare directly N-partite ($N > 2$) optical entangled states in a very compact device, and provides huge scaling potential [20,21].

On the other hand, for implementing a CV quantum information network, it is also necessary to find a scheme which can control and enhance the generated multipartite entangled states. In fact, due to the unavoidable intracavity losses in OPAs and the limitation of the effective nonlinear coefficient of a crystal, the squeezing and the entanglement of the quantum states produced by a single OPA is not high enough under usual conditions, thus the enhancement of the squeezing and entanglement is especially desired. Using a phase-sensitive DOPA (NOPA) the manipulation and the enhancement of a squeezed vacuum field (bipartite entangled optical beams) have been experimentally demonstrated, in which the manipulation is achieved via a second-order nonlinear interaction inside an optical cavity [22,23]. Yanagisawa *et al.* and Gough *et al.* proposed an enhancement scheme of optical field squeezing by placing a linear optical component in a loop in a simple coherent feedback mechanism involving a beam splitter [24,25]. The theoretical proposal was experimentally realized by Furusawa' group very recently [26]. Differentiating from the usual measurement-based control for quantum systems [27,28], the nonmeasurement-based coherent feedback control (CFC) is a control method without any backaction noises induced by the measurement. Thus, CFC is suitable to be used in CV quantum information processing, where any excess noises will reduce its precision [24,25]. Typically this control method is specially appropriate to be applied

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for enhancing squeezing and entanglement of optical fields. Reference [26] reports the first experimental demonstration of CFC on squeezed optical field, in which the squeezing degree of a single-mode squeezed state is enhanced. The CFC model is completely linear and can work in the case of quantum optics. To the best of our knowledge, there is no theoretical and experimental presentation to discuss the enhancement and manipulation of CV multipartite entanglement of optical fields. So far, the theoretical [24,25] and the experimental [26] demonstration of the ability of CFC on manipulating squeezing shows that the linear optical CFC loop is able to control the quantum fluctuations of the amplitude and phase quadratures of optical fields, which motivates us to explore the possible effect of CFC on CV multipartite entanglement.

In this paper we propose a CFC-NOPA system to achieve the generation and the manipulation of multipartite entangled states of light, simultaneously. The multipartite entangled states originally produced by the NOPA are injected into the CFC loop directly, in which the entangled optical beams are split into two parts by a beam splitter. One of the two parts is fed back to the NOPA and the other part serves as the final output multipartite entangled states of optical fields from this system. The calculated results show that the features of the CV multipartite entanglement of the output optical fields can be controlled by tuning the transmissivity of the beam splitter, that is, by adjusting the feedback amount of the entangled optical fields returned into NOPA. In this way, the CFC loop coherently controls the original output of the NOPA and feeds a part of them back into the NOPA to control its performance. The tunable optical beam splitter in the CFC loop is named the coherent feedback (CF) controller. The physical conditions for realizing the multipartite entanglement enhancement are found by means of numerical calculations based on scalable experimental parameters. In the following we will describe the CFC-NOPA system first in Sec. II. Then the operation principle of NOPA and the corresponding mathematic expressions for the generation of the multipartite CV entanglement are introduced in Sec. III. In Sec. IV the manipulation effects of CFC loop of the multipartite entangled optical fields generated by NOPA are analyzed. Finally, a brief conclusion is given in Sec. V.

II. SCHEMATIC OF CFC-NOPA SYSTEM

Since there are unavoidable losses in any real NOPAs, the experimentally obtainable multipartite entanglement degree from a single NOPA cannot be high enough under general conditions. The performance of a simple CFC linear optical loop on manipulating and enhancing single-mode squeezing has been theoretically and experimentally proved [24–26]. Our aim is to directly link a NOPA, which generates optical multipartite entangled states, and a CFC loop, which feeds a part of originally entangled optical fields back into the NOPA. We found that the multipartite entanglement of the final output optical fields from the CFC-NOPA system can be efficiently enhanced under appropriate conditions and the entanglement degree can be controlled by simply changing the transmissivity of the CF controller.

The CFC-NOPA system is depicted in Fig. 1. The system consists of two parts: (1) a NOPA as the source of the

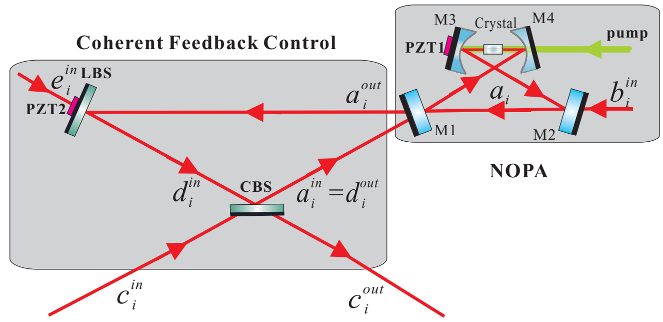

FIG. 1. (Color online) Schematic of CFC-NOPA system.

multipartite optical entangled states and (2) a CFC loop for implementing the manipulation and enhancement of multipartite entangled states. The NOPA has a bow-tie type ring configuration consisting of a nonlinear crystal, two flat mirrors $M_{1(2)}$, and two spherical mirrors $M_{3(4)}$. The input and output mirror M_1 has partial transmission and the other three cavity mirrors are highly reflective for the subharmonic optical field, and all the four mirrors are antireflective for the harmonic pump field. The strong harmonic-wave pump field is regarded as a classical field and does not resonate in the optical cavity. M_3 is mounted on a piezoelectric transducer (PZT_1) for scanning or locking the cavity length of the NOPA to the resonance with the subharmonic field. The input optical modes $\hat{a}_i^{\text{in}}$ ($i = 1, 2, \ldots, N$) are coupled to the intracavity optical modes $\hat{a}_i$ through the input and output coupler mirror M_1 with transmissivity efficiency γ_1. All other losses are modeled as the unwanted vacuum fields $\hat{b}_i^{\text{in}}$, which are coupled to the intracavity optical modes $\hat{a}_i$ through mirror M_2 with the intracavity loss γ_2. Through locking the cavity length resonating with the injected optical modes $\hat{a}_i$ and the relative phase between $\hat{a}_i$ and the pump field, the output longitudinal modes $\hat{a}_i^{\text{out}}$ with different frequencies (the frequency difference between any two neighboring modes equals to the free spectrum range of the resonator) are entangled with each other via concurrent interactions in a second-order nonlinear medium inside an NOPA, which constitute the multipartite entangled states [21].

In the CFC loop, a control beam splitter (CBS) with tunable transmissivity t for the injected signal filed $\hat{c}_i^{\text{in}}$ plays both roles of a controller and an input-output port. The loss of the CF loop can be regarded as an unwanted vacuum noise $\hat{e}_i^{\text{in}}$ coupled from the lossy beam splitter (LBS) with the transmissivity l. The weak coherent optical input signal field $\hat{c}_i^{\text{in}}$ is injected in CFC from CBS and the CF loop is locked through the PZT_2 mounted on the LBS. Then the transmitted field $\sqrt{t}\hat{c}_i^{\text{in}}$ and the reflected field $\sqrt{1-t}\hat{d}_i^{\text{in}}$ from CBS together serves as the input field of NOPA ($\hat{a}_i^{\text{in}} = \hat{d}_i^{\text{out}} = \sqrt{t}\hat{c}_i^{\text{in}} + \sqrt{1-t}\hat{d}_i^{\text{in}}$). The output multipartite entangled optical field $\hat{a}_i^{\text{out}}$ from the NOPA is reflected by LBS and then becomes the incident field of CBS. The final output field $\hat{c}_i^{\text{out}}$ of the CFC-NOPA system includes the transmitted field $\sqrt{t}\hat{d}_i^{\text{in}}$ and the reflected field $\sqrt{1-t}\hat{c}_i^{\text{in}}$. We have

$$\begin{aligned}\hat{c}_i^{\text{out}} &= \sqrt{t}\hat{d}_i^{\text{in}} - \sqrt{1-t}\hat{c}_i^{\text{in}} \\ &= \sqrt{t(1-l)}\hat{a}_i^{\text{out}} + \sqrt{tl}\hat{e}_i^{\text{in}} - \sqrt{1-t}\hat{c}_i^{\text{in}} \qquad (1)\end{aligned}$$

and

$$\hat{d}_i^{\rm out} = \sqrt{t}\hat{c}_i^{\rm in} + \sqrt{1-t}\hat{d}_i^{\rm in} = \sqrt{t}\hat{c}_i^{\rm in} + \sqrt{(1-t)(1-l)}\hat{a}_i^{\rm out} + \sqrt{(1-t)l}\hat{e}_i^{\rm in}. \quad (2)$$

III. OPERATION PRINCIPLE OF NOPA FOR MULTIPARTITE ENTANGLEMENT GENERATION

According to the multipartite nonseparability criterion proposed by van Loock and Furusawa [29], if the quadrature correlations satisfy the inequality $\langle\Delta(\hat{X}_i - \hat{X}_j)^2\rangle + \langle\Delta[\sum_{i=1}^{N}(\hat{Y}_i)]^2\rangle < 4$ or $\langle\Delta[\sum_{i=1}^{N}(\hat{X}_i)]^2\rangle + \langle\Delta(\hat{Y}_i - \hat{Y}_j)^2\rangle < 4$ $(i,j = 1,2,\ldots,N)$, where $\hat{X}_i$ and $\hat{Y}_i$ are the quadrature amplitude and phase components, these entangled optical fields can be named as GHZ-like entangled states, which are a type of the multipartite entangled states. Based on the theoretical model in Ref. [21], we discuss the generation of multipartite entangled optical fields from an NOPA, and characterize the multipartite entanglement using above criterion. The interaction Hamiltonian of the system in the interaction picture is given by $H_{\rm sys} = i\hbar k\sum_{i=1}^{N}\sum_{j>i}^{N}(\hat{a}_i^\dagger\hat{a}_j^\dagger - \hat{a}_i\hat{a}_j)$, which corresponds to the NOPA operated at parametric amplification. $\hat{a}_i$ is the annihilation operator of an intracavity mode i in the NOPA. The nonlinear coupling efficiency $k = \beta\chi$ is proportional to the pump parameter $\beta = (p_{\rm pump}/p_{\rm th})^{1/2}$ ($p_{\rm pump}$ is the pump power and $p_{\rm th}$ is the threshold pump power of NOPA) and the nonlinear coupling coefficient χ of the medium which must simultaneously phase match several second-order nonlinearities. In this case, each pair of the generated subharmonic fields in the NOPA forms a two mode squeezed state. The round trip time of light in the cavity is represented by τ. The quantum Langevin motion equations of the intracavity optical fields $\hat{a}_i$ are given by

$$\tau\frac{d}{dt}\hat{a}_1 = k\hat{a}_2^\dagger + k\hat{a}_3^\dagger + \cdots + k\hat{a}_N^\dagger - (\gamma_1+\gamma_2)\hat{a}_1 + \sqrt{2\gamma_1}\hat{a}_1^{\rm in} + \sqrt{2\gamma_2}\hat{b}_1^{\rm in}, \quad (3a)$$

$$\tau\frac{d}{dt}\hat{a}_2 = k\hat{a}_1^\dagger + k\hat{a}_3^\dagger + \cdots + k\hat{a}_N^\dagger - (\gamma_1+\gamma_2)\hat{a}_2 + \sqrt{2\gamma_1}\hat{a}_2^{\rm in} + \sqrt{2\gamma_2}\hat{b}_2^{\rm in}, \quad (3b)$$

$$\cdots$$

$$\tau\frac{d}{dt}\hat{a}_N = k\hat{a}_1^\dagger + k\hat{a}_2^\dagger + \cdots + k\hat{a}_{N-1}^\dagger - (\gamma_1+\gamma_2)\hat{a}_N + \sqrt{2\gamma_1}\hat{a}_N^{\rm in} + \sqrt{2\gamma_2}\hat{b}_N^{\rm in}. \quad (3N)$$

The output and the intracavity optical fields satisfy the following boundary condition: $\hat{a}_i^{\rm out} = \sqrt{\gamma_1}\hat{a}_i - \hat{a}_i^{\rm in}$. In the linearized description of fields, the operators can be expressed by the sum of an average steady state value $\langle x_i\rangle(\langle y_i\rangle)$ and a fluctuating component $\delta\hat{x}_i(\delta\hat{y}_i)$, that is, $\hat{x}_i = \langle x_i\rangle + \delta\hat{x}_i$ $(\hat{y}_i = \langle y_i\rangle + \delta\hat{y}_i)$. Then we implement the Fourier transformation $\hat{O}(\Omega) = (1/2)^{1/2}\int dt\hat{o}(t)e^{-i\Omega T}$ with the canonical commutative relation $[\hat{O}(\Omega),\hat{O}(\Omega')] = \delta(\Omega-\Omega')$. The calculated quadrature correlation variances of the output optical fields originally produced by the NOPA equal to

$$\delta\hat{X}_{ai}^{\rm out} - \delta\hat{X}_{aj}^{\rm out} = m_1(\delta\hat{X}_{ai}^{\rm in} - \delta\hat{X}_{aj}^{\rm in}) + n_1(\delta\hat{X}_{bi}^{\rm in} - \delta\hat{X}_{bj}^{\rm in}), \quad (4)$$

$$\sum_{i=1}^{N}\delta\hat{Y}_{ai}^{\rm out} = m_2\sum_{i=1}^{N}\delta\hat{Y}_{ai}^{\rm in} + n_2\sum_{i=1}^{N}\delta\hat{Y}_{bi}^{\rm in}, \quad (5)$$

$$\sum_{i=1}^{N}\delta\hat{X}_{ai}^{\rm out} = m_3\sum_{i=1}^{N}\delta\hat{X}_{ai}^{\rm in} + n_3\sum_{i=1}^{N}\delta\hat{X}_{bi}^{\rm in}, \quad (6)$$

$$\delta\hat{Y}_{ai}^{\rm out} - \delta\hat{Y}_{aj}^{\rm out} = m_4(\delta\hat{Y}_{ai}^{\rm in} - \delta\hat{Y}_{aj}^{\rm in}) + n_4(\delta\hat{Y}_{bi}^{\rm in} - \delta\hat{Y}_{bj}^{\rm in}), \quad (7)$$

where $m_1 = (-k+\gamma_1-\gamma_2-i\omega\tau)/(k+\gamma_1+\gamma_2+i\omega\tau)$, $n_1 = (2\sqrt{\gamma_1\gamma_2})/(k+\gamma_1+\gamma_2+i\omega\tau)$, $m_2 = [-(n-1)k+\gamma_1-\gamma_2-i\omega\tau]/[(n-1)k+\gamma_1+\gamma_2+i\omega\tau]$, $n_2 = (2\sqrt{\gamma_1\gamma_2})/[(n-1)k+\gamma_1+\gamma_2+i\omega\tau]$, $m_3 = [(n-1)k+\gamma_1-\gamma_2-i\omega\tau]/[-(n-1)k+\gamma_1+\gamma_2+i\omega\tau]$, $n_3 = (2\sqrt{\gamma_1\gamma_2})/[-(n-1)k+\gamma_1+\gamma_2+i\omega\tau]$, $m_4 = (k+\gamma_1-\gamma_2-i\omega\tau)/(-k+\gamma_1+\gamma_2+i\omega\tau)$, and $n_4 = (2\sqrt{\gamma_1\gamma_2})/(-k+\gamma_1+\gamma_2+i\omega\tau)$. $\omega = 2\pi\Omega$ is the analysis frequency. $\hat{X}_{ai}^{\rm in}$ ($\hat{X}_{ai}^{\rm out}$) and $\hat{Y}_{ai}^{\rm in}$ ($\hat{Y}_{ai}^{\rm out}$) are the quadrature amplitude and phase of the ith input (output) mode of the NOPA, respectively.

IV. MANIPULATION OF THE CFC LOOP OF MULTIPARTITE ENTANGLED OPTICAL FIELDS

When an optical coherent state is injected into the CFC-NOPA system as $\hat{c}_i^{\rm in}$, by solving the equations of the quadrature correlation variances of the originally optical fields produced by the NOPA operated at the parametric amplification [Eqs. (4)–(7)] and using the input-output relations of the CBS [Eqs. (1) and (2)], we obtain the quantum correlation variances

$$\begin{aligned}\langle\Delta(\delta\hat{X}_i - \delta\hat{X}_j)^2\rangle + \left\langle\left(\Delta\sum_{i=1}^{N}\delta\hat{Y}_i\right)^2\right\rangle &= 2[-n_1\sqrt{st} + (m_1n_1s\sqrt{tr})/(-1+m_1\sqrt{sr})]\\ &\quad+2[\sqrt{r} + (m_1t\sqrt{s})/(-1+m_1\sqrt{sr})]\\ &\quad+2[-\sqrt{lt} + (m_1\sqrt{slrt})/(-1+m_1\sqrt{sr})]\\ &\quad+N[-n_2\sqrt{st} + (m_2n_2s\sqrt{tr})/(-1+m_2\sqrt{sr})]\\ &\quad+N[\sqrt{r} + (m_2t\sqrt{s})/(-1+m_2\sqrt{sr})]\\ &\quad+N[-\sqrt{lt} + (m_2\sqrt{slrt})/(-1+m_2\sqrt{sr})],\end{aligned} \quad (8)$$

where r and s are defined as $r = 1-t$ and $s = 1-l$, respectively. Equation (8) shows that the quantum correlation variances characterizing the multipartite entanglement of the final output optical fields from the CFC-NOPA system depend on not only the parameters of NOPA (the construction parameters of optical cavity, the pump parameter, and the analysis frequency), but also the transmissivity of the CF controller and the losses of the feedback loop. It means that the features of the multipartite entangled states originally generated by the NOPA can be controlled by the attached CFC loop. Since the function expression in Eq. (8) is relatively complex, we will respectively study the dependence of the correlation variances on the changeable parameters t, ω, and β for a system with given construction parameters by means of the numerical calculations. In order to provide useful references for practical system designs, all parameter values are experimentally reachable. For the simplicity and without losing the generality, we numerically calculate the performance of the CFC-NOPA system for the quadripartite

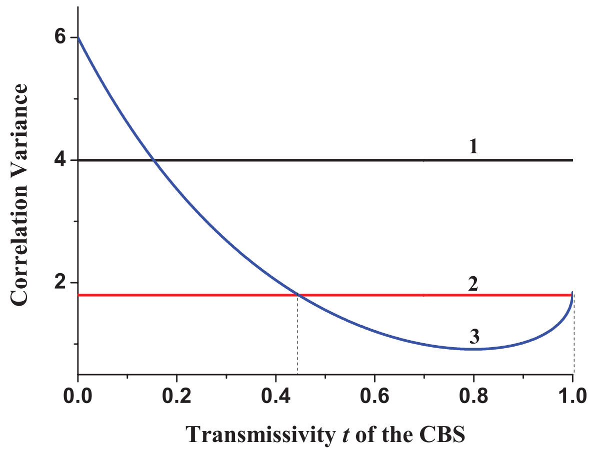

FIG. 2. (Color online) Dependence of the correlation variances $\langle\Delta(\delta\hat{X}_i - \delta\hat{X}_j)^2\rangle + \langle(\Delta\sum_{i=1}^{4}\delta\hat{Y}_i)^2\rangle$ on the transmissivity t of the CBS. 1: QNL, 2: NOPA only, and 3: CFC-NOPA.

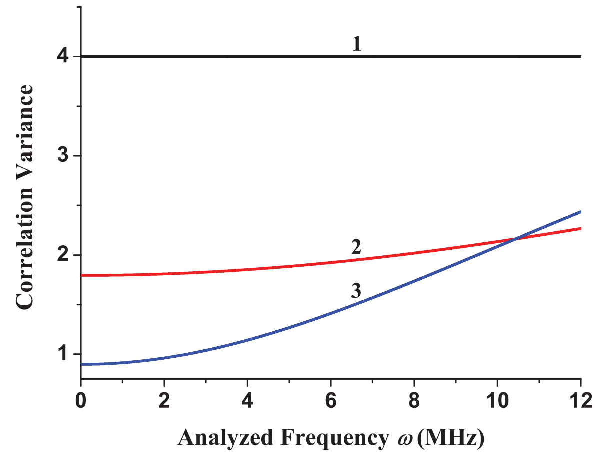

FIG. 3. (Color online) Dependence of the correlation variances $\langle\Delta(\delta\hat{X}_i - \delta\hat{X}_j)^2\rangle + \langle(\Delta\sum_{i=1}^{4}\delta\hat{Y}_i)^2\rangle$ on the analyzed frequency ω. 1: QNL, 2: NOPA only, and 3: CFC-NOPA.

entanglement generation ($N = 4$). The parameter values used in the calculation are as following: the input-output couple efficiency γ_1 is 0.1, the loss of the intracavity γ_2 is 0.003, the round trip time τ of light in the cavity is 6.7×10^{-10} s, and the CF loop loss l is 0.01. First, for a given low analysis frequency ($\omega = 1$ MHz) and a weak pumping strength ($\beta = 0.15$), we investigate the dependence of the quantum correlation variances $\langle\Delta(\delta\hat{X}_i - \delta\hat{X}_j)^2\rangle + \langle(\Delta\sum_{i=1}^{4}\delta\hat{Y}_i)^2\rangle$ on the transmissivity t of the CF controller while other parameters keep unchanging. According to the theoretical estimate the quantum correlations of the amplitude and phase quadratures among the multipartite entangled optical modes originally produced by the NOPA are better under the lower analysis frequency and the weaker pump [30–32]. For general continuous-wave pump laser the intensity and phase fluctuations are quite strong around zero frequency and then reduce gradually. Usually the fluctuations can achieve the quantum noise limit after the frequency is higher than 1 MHz if the pump laser is filtered by the mode cleaner with a high finesse [33]. For the pump strength of $\beta < 1$, the NOPA operates below its oscillation threshold and thus is stable. Considering these experimental conditions, we take $\omega = 1$ MHz, and $\beta = 0.15$ in the example for the numerical calculation. Of course, as a theoretical model Eq. (8) can be used for calculating the quantum correlation variances of the output field of CFC-NOPA systems with any chosen parameters. From Fig. 2 it can be seen that the entanglement level of the CFC loop (line 3) is higher or lower than that without the use of CFC (line 2) for different values of t. For the case of $t = 0$, the injected coherent states are totally reflected by CBS and there is no entangled light produced by the NOPA to be transmitted, thus the corresponding correlation variances involve six vacuum noises, that is, should be $\langle\Delta(\delta\hat{X}_i - \delta\hat{X}_j)^2\rangle + \langle(\Delta\sum_{i=1}^{4}\delta\hat{Y}_i)^2\rangle = 6$, which is higher than the limitation of the inseparability criterion for the quadrature CV entanglement (four vacuum noises) [29]. In fact, two parts are involved in the final output field and the field fed back into the NOPA when $0 < t < 1$. One part is the multipartite entangled light, which plays the positive role for the entanglement enhancement. The other part is the input coherent light and the excess noise resulting from the CFC loop, which reduces the entanglement. For the case of $t < 0.45$, the positive role of the multipartite entangled light for enhancing entanglement is smaller than the negative influence of the input coherent light and the excess noises, thus the correlation variances on line 3 are higher than that without using the CFC loop (line 2). After $t > 0.45$, the positive role of the coherent feedback surpasses the negative influence, so the quantum correlations of the final output field (line 3) become better than that without the use of CFC (line 2). Until $t = 0.8$ the effect of the positive role reaches the optimal situation, and when $t > 0.8$ the ratio of the negative influence of the excess noises increases which results in the correlation variances raising again. In the range of $0.45 < t < 1$, the positive role of the CFC is stronger than the negative influence of the injected excess noises, so the multipartite entanglement of the final output fields is enhanced than that originally produced by the NOPA, and at $t = 0.8$ the optimal entanglement is obtained. When $t = 1$, the CFC-NOPA is operated at the situation without the feedback and thus line 3 and line 2 nearly overlap at this point. Figure 2 shows that the quantum correlation variances among the amplitude and phase quadratures of the multipartite entangled state can be controlled simply by tuning the transitivity t of a CF controller. For a set of given system parameters, one can find the transmissivity range of entanglement enhancement and the optimal t by the numerical calculation based on Eq. (8).

The spectrum distribution of the correlation variances is shown in Fig. 3, where all system parameters are the same as that used in Fig. 2 and $t = 0.8$ is chosen. When the analysis frequency is lower than 10.4 MHz, the entanglement is enhanced by the CFC (line 3 is below line 2). At the zero frequency the entanglement enhancement reaches the maximum, and when the frequency is higher than 10.4 MHz the entanglement becomes worse. From Fig. 3 we can see that the frequency dependence of the correlation variances of the output optical fields from the CFC-NOPA system (line 3) is stronger than that of the entangled light originally produced by the NOPA (line 2). That is because the delay of the light in the feedback loop will affect the control performance and the operation bandwidth of the CFC-NOPA system,

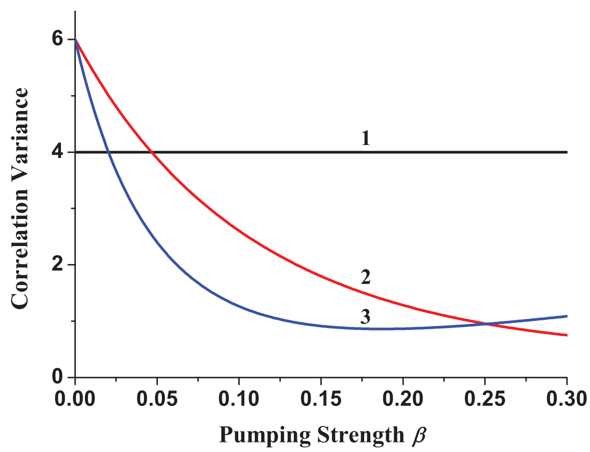

FIG. 4. (Color online) Dependence of the correlation variances $\langle\Delta(\delta\hat{X}_i-\delta\hat{X}_j)^2\rangle+\langle(\Delta\sum_{i=1}^{4}\delta\hat{Y}_i)^2\rangle$ on the pumping strength β of the NOPA. 1: QNL, 2: NOPA only, and 3: CFC-NOPA.

and the influence becomes stronger at the region of high frequencies [26].

We also calculate the dependence of the correlation variances on the pumping strength β of the NOPA at $t=0.8$ and $\omega=1$ MHz. Figure 4 shows that the control action of the CF controller on the multipartite entangled states produced by the NOPA depends upon the pump power of the NOPA. In the CFC-NOPA system the part of the multipartite entangled fields fed back into the NOPA enhance the capacity of generating entanglement via the nonlinear interaction inside the NOPA, but at the same time the excess losses in the CFC loop is also fed into the NOPA which reduce the entanglement of the generated quantum states. When the pump power is increased both the positive and the negative influences are simultaneously raised. At lower pump power the positive effect is stronger, thus the entanglement is enhanced by the CFC loop. But when the pump power is higher ($\beta>0.25$), the negative effect becomes dominant and the entanglement is reduced. There is a trade-off between the entanglement enhancement effect due to the parametric interaction inside NOPA and the opposite influence induced by the CF loop losses. For a given system we have to find the range of the optimal pump powers to achieve the best entanglement enhancement (In our case it is about $0.125<\beta<0.225$). The similar pumping dependence of single-mode squeezing in a CFC-NOPA system has been experimentally proved in Ref. [26].

V. CONCLUSION

In summary, we have theoretically proposed a CFC loop of multipartite entangled optical fields. The dependencies of the correlation variances of the multipartite entangled states produced by the CFC-NOPA system on the transmissivity of CFC, the analysis frequency, and the pumping strength are numerically calculated. The calculated results show that the multipartite entanglement of the output fields from a NOPA can be controlled by a pure coherent feedback mechanism. The correlation variances among the amplitude and phase quadratures of the multipartite entangled fields produced by a NOPA can be manipulated only by simply tuning the transmissivity of a CF controller. For a given NOPA we may choose the optimal transmissivity t of the CBS and the pumping strength β to achieve the possibly largest entanglement enhancement. Besides the capacity of the entanglement enhancement, the CFC scheme can tune the correlations of the quantum fluctuations among the submodes of a multipartite entangled optical fields only by means of simply linearly optical operation, which has potential applications in the future CV quantum information processing and communication networks.

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